

APPLICATION OF VIM, HPM AND CM TO THE SYSTEM OF STRONGLY NONLINEAR FIN PROBLEM

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ABSTRACT

The nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient is analytically studied. Collocation method (CM), Variation iteration method (VIM) and Homotopy Perturbation Method (HPM) are used to solve the present problem. Also, fourth order Runge–Kutta numerical method is applied as a numerical method for validation. Analytical results are presented through the graphs and the tables in various values of parameters. The results reveal that the CM is very effective, simple and more accurate than other techniques. Furthermore, we analyze the effects of some physical applicable parameters in this problem such as thermal conductivity parameter (β), thermo-geometric fin parameter (M) and heat transfer mode (m).

KEYWORDS: *Collocation method (CM); Variational iteration method (VIM); Homotopy Perturbation Method (HPM); Heat transfer; Fin*

1.0 INTRODUCTION

Fins or extended surfaces are frequently used to enhance the heat transfer between a solid surface and its surrounding medium. Extended surfaces are extensively used in various industrial applications; for example, liquid-gas heat exchangers, air-cooled internal combustion engines, the electrical apparatus, nucleate boiling, etc. Since most of problems and scientific phenomenon such as heat transfer problems for the fins (Kiwari, 2007; Gorla Reddy & Bakier, 2011; Domairry & Fazeli, 2011; Ganji, 2011; Khani et al, 2009), are inherently of nonlinearity. Therefore, the differential equation for a convective fin does not admit an exact solution and these nonlinear equations should be solved using other methods, Such as numerical analysis or analytical method. In the analytical perturbation method, we should exert a small parameter in the equation. Therefore, finding this parameter and exerting it into the equation is difficulties of this method. In recent years, scientists have presented some new methods for solving nonlinear differential equations; for instance, δ -expansion method (Ganji & Hashemi, 2011a), Adomian's decomposition method (Ganji & Hashemi, 2011b), Homotopy perturbation method (HPM) (He, 1999; He 2005a; He, 2005b; Esmailpour & Ganji, 2007; He, 2010; Ganji & Rajabi, 2006; Ganji, Ganji & Ganji, 2011) and Variational iteration method (VIM) (He, 2007; He & Wu, 2006; Ganji, Rostamiyan, Petroudi & Nejad, 2014; Ganji, Tari & Jooybari, 2014; He, 1999). One of the other semi-exact methods is the weighted residual methods (WRMs). Collocation method (CM), Galerkin method (GM), and least square method (LSM) are examples of WRM. These methods are the most effective and

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convenient ones for both linear and nonlinear equations. Stern and Rasmussen used collocation method for solving a third order linear differential equation (Stern and Rasmussen, 1996). Hu and Li and Herrera et al. applied collocation method for Poisson's equation and advection–diffusion equation respectively (Hu & Li, 2006; Herrera, Diazviera, Yates, 2004). Arnau et al., presented a new alternative method based in polynomial collocation and used it to model the flow in intake and exhaust of internal combustion engine. Hendi and Albugami solved Fredholm–Volterra integral equation using collocation method (Hendi & Albugami, 2010). Recently Hatami and Ganji applied CM on non-Newtonian nanofluid passing through the porous media between two coaxial cylinders (Hatami and Ganji, 2013). In this article, the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient is solved through the three methods: Collocation method, Homotopy perturbation method and the Variation iteration method. Also Runge-Kutta method is used to evaluate the excellence and accuracy of the proposed methods. The computations show that CM allows us to obtain approximations with an error relative to the numerical solution smaller than the errors obtained using other methods.

2.0 METHODOLOGY

2.1 Governing equations

Consider a straight fin with an arbitrary constant cross-sectional area A_c ; perimeter p and length L . The fin is attached to a base surface of temperature T_b , extends into a fluid of temperature T_a , and its tip is insulated. The one-dimensional energy balance equation is given:

$$A_c \frac{d}{dX} \left(k \frac{dT}{dX} \right) + p h (T - T_a) = 0, \quad 0 < X < L \quad (1)$$

The thermal conductivity of the fin material is assumed to be a linear function of temperature according to:

$$k = k(T) = k_a [1 + \lambda (T - T_a)] \quad (2)$$

where k_a is the thermal conductivity at the ambient fluid temperature of the fin and λ is the parameter describing the thermal conductivity variation. Also It is considered that h vary with temperature function by the following equations:

$$h = h(T) = h_b \left(\frac{T - T_a}{T_b - T_a} \right)^m \quad (3)$$

where h_b is the heat transfer coefficient at the base temperature. The exponent m depends on the heat transfer mode. Typical values of n are $-1/4$ for laminar film boiling or

condensation, 0 constant for heat transfer coefficient, $1/4$ for laminar natural convection, $1/3$ for turbulent natural convection, 2 for nucleate boiling, and 3 for radiation. In order to simplify the energy equation, the dimensionless parameters are defined as follows:

$$\theta = \frac{T - T_a}{T_b - T_a}, \quad x = \frac{X}{L}, \quad k = \frac{k}{k_a}, \quad \beta = \lambda(T_b - T_a), \quad M^2 = \frac{h_b p l^2}{k_a A_c} \quad (4)$$

Hence, the energy Equation (1) will take the form (Khani, Ahmadzadeh Raji, Hamed Nejad, 2009):

$$\frac{d}{dX} \left[k(\theta) \frac{d\theta}{dX} \right] = M^2 \theta^{m+1}, \quad 0 < x < 1 \quad (5)$$

The boundary conditions are:

$$\begin{aligned} \theta'(0) &= 0 && \text{at the tip,} \\ \theta(1) &= 1 && \text{at the base} \end{aligned} \quad (6)$$

The governing Equation (5) can be rewritten into the following form:

$$(1 + \beta \theta) \theta'' - M^2 \theta^{m+1} + \beta (\theta')^2 = 0 \quad (7)$$

2.2 Basic idea of Collocation method

Collocation method is one of the approximation techniques for solving differential equations called the Weighted Residual Methods (WRMs). For the conception of the main idea of this method, suppose a differential operator D is acted on a function u to produce a function p (Hatami and Ganji, 2013):

$$D(u(x)) = p(x) \quad (8)$$

We wish to approximate u by a function \tilde{u} , which is a linear combination of basic functions chosen from a linearly independent set. That is:

$$u = \tilde{u} = \sum_{i=1}^n c_i \varphi_i \quad (9)$$

Now, when substituted into the differential operator, D , the result of the operations is not $p(x)$. Hence an error or residual will exist:

$$E(x) = R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \quad (10)$$

The notion in the Collocation is to force the residual to zero in some average sense over the domain. That is:

$$\int_x R(x) W_i(x) dx = 0, \quad i = 1, 2, \dots, n \quad (11)$$

where the number of weight functions w_i is exactly equal the number of unknown constants c_i in \tilde{u} . The result is a set of n algebraic equations for the unknown constants

c_i . For Collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is, $w_i(x) = \delta(x - x_i)$. The Dirac δ function has the property of:

$$\delta(x - x_i) = \begin{cases} 1 & \text{if } x = x_i \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

The residual function in Equation (10) must be forced to be zero at specific points.

2.3 Application of collocation method

We wish to obtain an approximate solution for this problem in the interval $0 < x < 1$.

To construct a trial solution θ , we choose the basic function to polynomial in x . The trial solution contains four undetermined coefficients and satisfies the conditions for all values of c as follows:

$$\theta(x) = 1 + c_1(1 - x^2) + c_2(1 - x^3) + c_3(1 - x^4) + c_4(1 - x^5) \quad (13)$$

Whereas the trial solution satisfies the boundary condition of Equation (6). The accuracy of the solution can be improved by increasing the number of its terms. When θ is introduced into differential equation it yields residual $R(x)$ as follows:

$$\begin{aligned} R(x) = & -2c_1 + 6\beta c_1^2 x^2 - 20\beta c_4^2 x^3 + 28\beta c_3^2 x^6 - 2\beta c_3 c_1 - 2\beta c_4 c_1 - 6\beta c_2 x \\ & - 12c_3 x^2 - M^2 \left(1 + c_1 - c_1 x^2 + c_2 - c_2 x^3 + c_3 - c_3 x^4 + c_4 - c_4 x^5 \right)^m c_2 + \dots \\ & - M^2 \left(1 + c_1 - c_1 x^2 + c_2 - c_2 x^3 + c_3 - c_3 x^4 + c_4 - c_4 x^5 \right)^m c_4 x^5 \end{aligned} \quad (14)$$

Now the problem of finding approximate solution of the problem in the interval $0 < x < 1$ becomes one adjusting the values of c_1, c_2, c_3 and c_4 . So that residual stays close to zero throughout the interval $0 < x < 1$. The basic assumption is that the residual does not deviate much from zero between collocation locations. For reaching to this aim, four specific points should be chosen. These points are:

$$R\left(\frac{1}{5}\right) = 0, \quad R\left(\frac{2}{5}\right) = 0, \quad R\left(\frac{3}{5}\right) = 0, \quad R\left(\frac{4}{5}\right) = 0, \quad (15)$$

$$\begin{aligned} R\left(\frac{1}{5}\right) = & -2c_1 - \frac{6}{5}c_2 - \frac{12}{25}c_3 - \frac{4}{25}c_4 - \frac{304}{125}\beta c_3 c_1 - \frac{6708}{3125}\beta c_4 c_1 - \frac{76}{25}\beta c_2 c_1 \\ & - \frac{21194}{15625}\beta c_2 c_4 - \frac{124}{125}M^2 \left(1 + \frac{24}{25}c_1 + \frac{124}{125}c_2 + \frac{624}{625}c_3 + \frac{3124}{3125}c_4 \right)^m c_2 + \dots \\ & - \frac{12491}{78125}\beta c_4^2 - M^2 \left(1 + \frac{24}{25}c_1 + \frac{124}{125}c_2 + \frac{624}{625}c_3 + \frac{3124}{3125}c_4 \right)^m = 0 \end{aligned} \quad (16)$$

$$\begin{aligned}
R\left(\frac{2}{5}\right) = & -2c_1 - \frac{12}{5}c_2 - \frac{48}{25}c_3 - \frac{32}{25}c_4 - \frac{394}{125}\beta c_3c_1 - \frac{8906}{3125}\beta c_4c_1 - \frac{78}{25}\beta c_2c_1 \\
& - \frac{53916}{15625}\beta c_2c_4 - \frac{117}{125}M^2\left(1 + \frac{21}{25}c_1 + \frac{117}{125}c_2 + \frac{609}{625}c_3 + \frac{3093}{3125}c_4\right)^m c_2 + \dots \\
& - \frac{97696}{78125}\beta c_4^2 - M^2\left(1 + \frac{21}{25}c_1 + \frac{117}{125}c_2 + \frac{609}{625}c_3 + \frac{3093}{3125}c_4\right)^m = 0
\end{aligned} \tag{17}$$

$$\begin{aligned}
R\left(\frac{3}{5}\right) = & -2c_1 - \frac{18}{5}c_2 - \frac{108}{25}c_3 - \frac{108}{25}c_4 - \frac{304}{125}\beta c_3c_1 - \frac{9544}{3125}\beta c_4c_1 - \frac{32}{25}\beta c_2c_1 \\
& - \frac{82926}{15625}\beta c_2c_4 - \frac{98}{125}M^2\left(1 + \frac{16}{25}c_1 + \frac{98}{125}c_2 + \frac{544}{625}c_3 + \frac{2882}{3125}c_4\right)^m c_2 + \dots \\
& - \frac{278451}{78125}\beta c_4^2 - M^2\left(1 + \frac{16}{25}c_1 + \frac{98}{125}c_2 + \frac{544}{625}c_3 + \frac{2882}{3125}c_4\right)^m = 0
\end{aligned} \tag{18}$$

$$\begin{aligned}
R\left(\frac{4}{5}\right) = & -2c_1 - \frac{24}{5}c_2 - \frac{192}{25}c_3 - \frac{256}{25}c_4 - \frac{326}{125}\beta c_3c_1 - \frac{4758}{3125}\beta c_4c_1 - \frac{86}{25}\beta c_2c_1 \\
& - \frac{5624}{15625}\beta c_2c_4 - \frac{61}{125}M^2\left(1 + \frac{9}{25}c_1 + \frac{61}{125}c_2 + \frac{369}{625}c_3 + \frac{2101}{3125}c_4\right)^m c_2 + \dots \\
& - \frac{210176}{78125}\beta c_4^2 - M^2\left(1 + \frac{9}{25}c_1 + \frac{61}{125}c_2 + \frac{369}{625}c_3 + \frac{2101}{3125}c_4\right)^m = 0
\end{aligned} \tag{19}$$

Thus we can obtain coefficient for different value of β , M and m . For example, Using Collocation method with $\beta=0.1, M=1, m=2$, $\theta(x)$ is as follows:

$$\theta(x) = 0.7634481393 + 0.2024689517x^2 + 0.0141005165x^3 + 0.0028024632x^4 + 0.0171799293x^5 \tag{20}$$

2.4 Variational iteration method

To illustrate the basic idea of variational iteration method, we consider the following general nonlinear system:

$$Lu + Nu = g(t) \tag{21}$$

where L is a linear operator, N nonlinear operator, $g(t)$ a homogeneous term. According to the variational iteration method, we can construct the following iteration formulation:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda [Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)] d\tau \tag{22}$$

where λ is a general Lagrangian multiplier, which can be identified optimally via the variational theory. The subscript n indicates the n th approximation and \tilde{u}_n is considered as a restricted variation, i.e., $\delta\tilde{u}_n = 0$.

2.5 Application of Variational iteration method

First we construct a correction functional which reads:

$$\theta_{n+1}(x) = \theta_n(x) + \int_0^x \lambda \left\{ [1 + \beta\theta_n(\tau)]\theta_n''(\tau) - M^2\theta_n^{m+1}(\tau) + \beta[\theta_n'(\tau)]^2 \right\} d\tau \tag{23}$$

Now we start with an arbitrary initial approximation that satisfies the initial condition. For example, when $M=1$, $\beta=-0.4$ and $m=0$, $\theta_0(x)$ is as follows:

$$\theta_0(x) = \frac{e^{-x}}{e^{-1}+e} + \frac{e^{-x}}{e^{-1}+e} \quad (24)$$

Also, making the above correction functional stationary, we can obtain following stationary conditions:

$$\lambda''(t) - \lambda(t) = 0, \quad 1 - \lambda'(t)|_{t=x} = 0, \quad \lambda(t)|_{t=x} = 0 \quad (25)$$

The Lagrangian multiplier can therefore be identified as:

$$\lambda = -\frac{1}{2} \left(e^{(x-\tau)} - e^{(\tau-x)} \right) \quad (26)$$

Substituting $\theta_0(x)$ and λ into Equation (23) and after some simplifications, we have:

$$\begin{aligned} \theta_1(x) = c_0 & \left[0.1192029221e^{-x+1} + 0.1192029221e^{x+1} - 0.003337477915e^x \right. \\ & - 0.003337477915e^{x+2} - 0.003337477915e^{-x} + 0.002503108436e^{2x} - 0.003337477915e^{-x+2} \\ & \left. - 0.003337477915e^{2x} + 0.003337477915e^{2x+2} + 0.003337477915e^{-2x} + 0.003337477915e^{-2x+2} \right] \end{aligned} \quad (27)$$

where $c_0 = \frac{1}{A}$, that $A = 1.124262852$

$$\begin{aligned} \theta_2(x) = c_1 & \left[-0.0000035250 \sinh(x)x + 0.106027626 e^{-x+1} + 0.106027626 e^{x+1} \right. \\ & - 0.002968592 e^x - 0.002968592 e^{x+2} - 0.002968592 e^{-x} - 0.002968592 e^{-x+2} \\ & \left. + \dots 0.000141638 \cosh(3x+3) + 0.000141638 \cosh(3x-3) - 0.00000396564 \cosh(3x+4) \right] \end{aligned} \quad (28)$$

where $c_1 = \frac{1}{B}$, that $B = 1.031737135$

In a similar manner, we will obtain other solutions for different cases M , β and m . The results are presented graphically.

2.6 Analysis of He's Homotopy perturbation method

To explain the basic ideas of this method, consider the following equation;

$$A(u) - f(r) = 0, \quad r \in \Omega \quad (29)$$

with the boundary condition of:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma \quad (30)$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω . A can be divided into two parts, which are L and N , where L is linear and N is nonlinear. Equation (1) can therefore be rewritten as follows;

$$L(u) + N(u) - f(r) = 0, r \in \Omega \quad (31)$$

Homotopy perturbation structure is shown as follows;

$$H(v, p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \quad (32)$$

where

$$v(r, p): \Omega \times [0, 1] \rightarrow R \quad (33)$$

In Equation (4), $p \in [0, 1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. It can be assumed the solution of Equation (4) can be written as a power series in p , as follows;

$$v = v_0 + p^1 v_1 + p^2 v_2 + \dots = \sum_{i=0}^n v_i p^i \quad (34)$$

and the best approximation for the solution is:

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + \dots \quad (35)$$

2.7 Application of Homotopy perturbation method

In this section, we will apply the HPM to nonlinear ordinary differential system (1). according to Equation (7), Using HPM, when $M = 1$, $\beta = -0.4$ and $m = 0$ leads to:

$$H(\theta, p) = (1-p)[\theta''(x) - \theta(x)] + p[(1-0.1\theta)\theta'' - \theta - 0.1(\theta')^2] = 0 \quad (36)$$

We consider $\theta(x)$ as following:

$$\theta(x) = \theta_0(x) + p\theta_1(x) + p^2\theta_2(x) \quad (37)$$

By substituting $\theta(x)$ from Equation (37) into Equation (36) and after some simplifications and rearrangements based on powers of p -terms, we have:

$$p^0: \quad \begin{aligned} \theta_0''(x) - \theta_0(x) &= 0 \\ \theta_0(1) &= 1, \theta_0'(0) = 0 \end{aligned} \quad (38)$$

$$p^1: \quad \begin{aligned} \theta_1''(x) - 0.4[\theta_0'(x)]^2 - \theta_1(x) - 0.4\theta_0(x)[\theta_0'(x)] &= 0 \\ \theta_1(1) &= 0, \theta_1'(0) = 0 \end{aligned} \quad (39)$$

$$p^2: \quad \begin{aligned} \theta_2''(x) - \theta_2(x) - 0.4\theta_1(x)\theta_0''(x) - 0.4\theta_0(x)\theta_1''(x) - 0.8\theta_1'(x)\theta_0'(x) &= 0 \\ \theta_2(1) &= 0, \theta_2'(0) = 0 \end{aligned} \quad (40)$$

Solving Equations (38) to (40) with boundary conditions, we have:

$$\theta_0(x) = \frac{e^{-x}}{e^{-1} + e} + \frac{e^x}{e^{-1} + e} \quad (41)$$

$$\theta_1(x) = -\frac{4}{15} \left[\frac{e^x(e^4 + 1)}{2e^{-1}e^2 + e^{-1} + e^{-1}e^4 + e^4 + 2e^2 + e} + \frac{e^{-x}(e^4 + 1)}{2e^{-1}e^2 + e^{-1} + e^{-1}e^4 + e^4 + 2e^2 + e} - \frac{e^{-2x+2}(e^4 + 1)}{e^4 + 30e^2 + 15} \right] \quad (40)$$

$$\theta_2(x) = \frac{1}{225} \left[\frac{e^{-x}(76e^5 + 5e + 5e^9 - 33e^7 - 9e^3)e^{-1}}{6e^{-1}e^2 + 4e^{-1}e^6 + e^{-1} + 4e^{-1}e^2 + e^{-1}e^8 + 4e^2 + e + ee^8 + 6ee^4 + 4ee^6} + \dots + \frac{e^{-3x+2}}{e^8 + 4e^2 + 1 + 6e^4 + 4e^6} \left[(-32e^x - 32e^{5x+4} + 27e^{6x+1} 12xe^{4x+1} + \dots - 12xe^{2x+3}) \right] \right] \quad (44)$$

The solution of this equation, when $p \rightarrow 1$, will be as follows:

$$\theta(x) = \theta_0(x) + \theta_1(x) + \theta_2(x) \quad (44)$$

In a similar manner, we will obtain other solutions for different cases M , β and m . The results are presented in next section.

3.0 RESULTS AND DISCUSSION

In this manuscript the collocation method such as analytical technique is employed to find an analytical solution of the nonlinear fin problem. The results are compared with other analytical methods such as VIM and HPM. For validation all these results are compared with the numerical solution. Figures 1-3 and Table 1 show the temperature distribution with the axial distance along the fin for the three methods. It is interesting to note that collocation method is very close to the numerical results and the results of HPM and VIM are significantly in error.

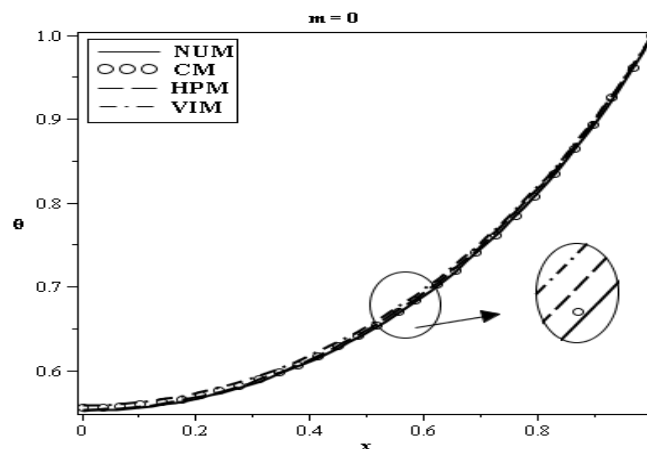


Figure 1. Comparison between the CM, VIM, HPM and numerical solution for $m = 0$, $M = 1$, $\beta = -0.4$

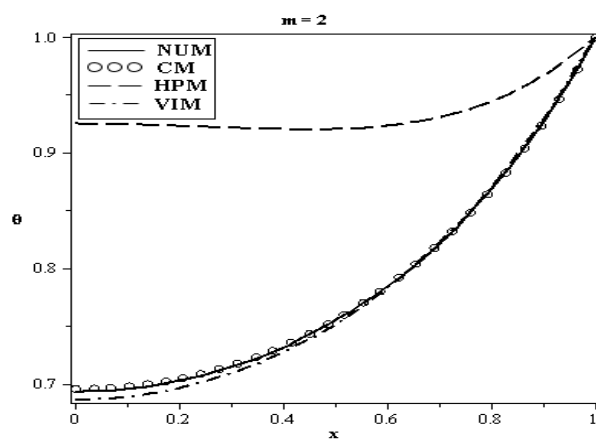


Figure 2. Comparison between the CM, VIM, HPM and numerical solution for $m = 2$, $M = 1$, $\beta = -0.4$

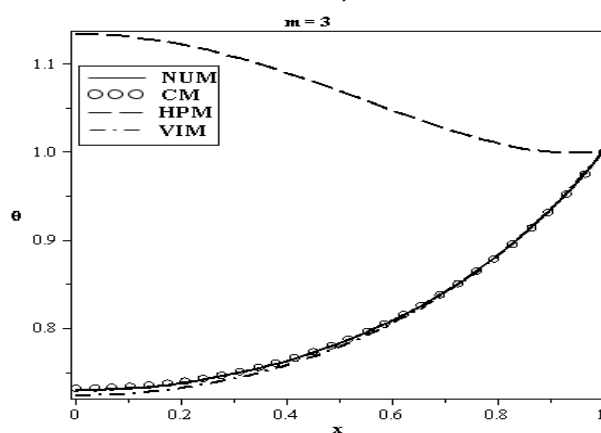


Figure 3. Comparison between the CM, VIM, HPM and numerical solution for $m = 3$, $M = 1$, $\beta = -0.4$

Table 1 The results of CM, VIM, HPM, and numerical methods for $m = 2$, $M = 1$, $\beta = -0.4$

| x | CM | VIM | HPM | NUM | ERROR VIM | ERROR HPM | ERROR CM |
|-----|------------|------------|------------|------------|--------------|--------------|-------------|
| 0 | 0.69636640 | 0.68655496 | 0.9250000 | 0.69340454 | 0.00684958 | 0.23159545 | 0.00296185 |
| 0.1 | 0.6984550 | 0.68900216 | 0.92451250 | 0.69571666 | 0.00671450 | 0.22879583 | 0.00273842 |
| 0.2 | 0.7050589 | 0.69641743 | 0.92320000 | 0.70271768 | 0.00630025 | 0.22048231 | 0.00234130 |
| 0.3 | 0.7165880 | 0.70902464 | 0.92151250 | 0.71460737 | 0.00558272 | 0.20690512 | 0.00198068 |
| 0.4 | 0.7334326 | 0.72720630 | 0.92020000 | 0.73173911 | 0.00453281 | 0.18846088 | 0.00169357 |
| 0.5 | 0.7561004 | 0.75151812 | 0.92031250 | 0.75465474 | 0.00313661 | 0.16565775 | 0.00144573 |
| 0.6 | 0.7853530 | 0.78271046 | 0.92320000 | 0.78414305 | 0.00143259 | 0.13905694 | 0.00121000 |
| 0.7 | 0.8223430 | 0.82175744 | 0.93051250 | 0.82133639 | 0.00042104 | 0.10917610 | 0.00100665 |
| 0.8 | 0.8687507 | 0.86989507 | 0.94420000 | 0.86787250 | 0.00202256 | 0.07632749 | 0.00087821 |
| 0.9 | 0.9269209 | 0.92866991 | 0.96651250 | 0.92617822 | 0.00249169 | 0.04033427 | 0.00074272 |

Figures 4, 5 and 6 show comparison between the numerical solutions and CM solutions in predicting θ for different values of thermal conductivity (β), thermo-geometric fin parameter (M) and heat transfer mode (m) respectively. We can see a very good agreement between the collocation and the numerical results. In addition, It can be seen in these figures that by increasing the β and m , velocity profiles increases, but the velocity profiles decreases with the increase in M .

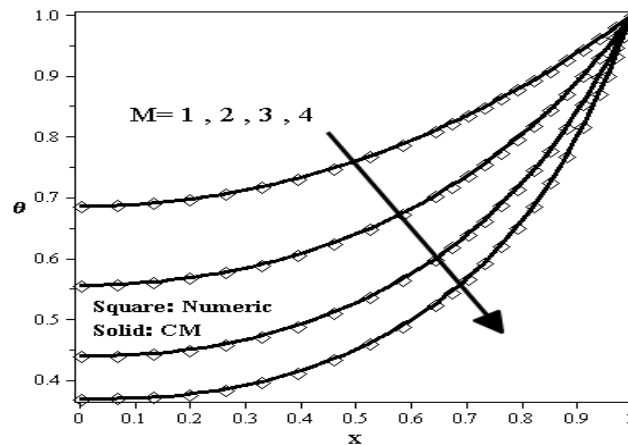


Figure 4. Effect of thermo-geometric fin parameter (M) on θ when $m=1/4$, $\beta=0.1$

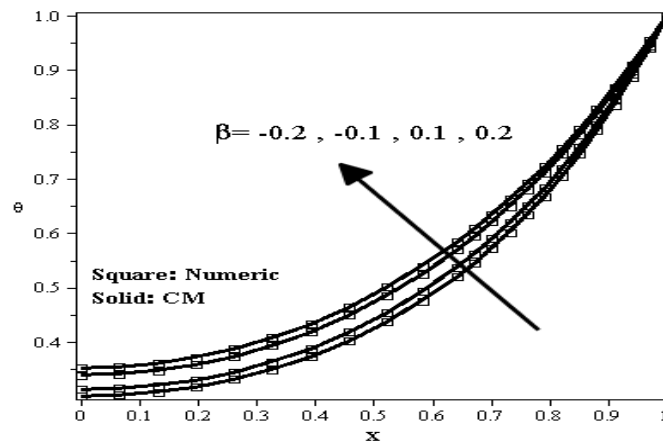


Figure 5. Effect of thermo-geometric fin parameter (β) on θ when $m=1/4$, $M=2$

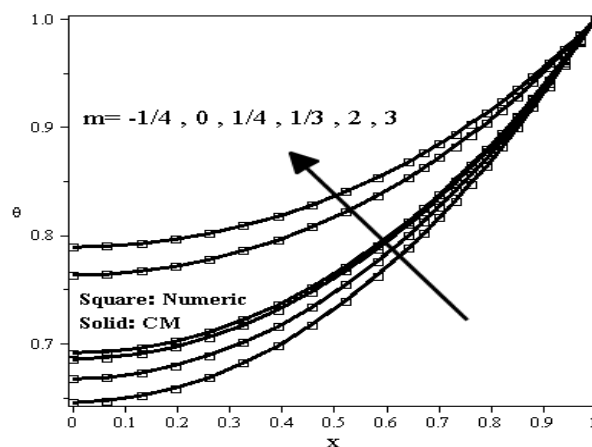


Figure 6. Effect of thermo-geometric fin parameter (β) on θ when $M=1$, $\beta=0.1$

4.0 CONCLUSION

In this paper, nonlinear heat transfer equations for fin with temperature-dependent thermal conductivity and heat transfer coefficient are presented and the CM has been successfully applied to find the most exact analytical solution. Furthermore, the obtained solutions by collocation method are compared with VIM, HPM and numerical solutions. The results demonstrate that the CM is powerful mathematical tools and has excellent agreement with numerical outcomes. Also accuracy of the solution can be increased by increasing the statements of the trial functions. The CM is very effective, simpler and offers superior accuracy compared with the variation iteration method and Homotopy Perturbation Method. It does not need any perturbation, linearization or small parameter versus Homotopy Perturbation Method and Variation iteration method.

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